Geometric Analysis of Shark Teeth
Worksheet

Shark teeth vary greatly in their geometry, and this variability in form determines their cutting performance. Variability in cutting performance affects their feeding ecology, and causality can therefore be established between the geometric variability of the teeth and the feeding ecology of the shark. A shark bite generally involves two behaviors, puncture and lateral head-shaking. Once the teeth have punctured into a prey item, the head is whipped from side to side (lateral head-shaking), sawing the teeth through the prey. The ability of a shark tooth to puncture a prey item is a function of the pressure generated along the surface of the tooth. However, the ability of a shark tooth to saw through prey during lateral head-shaking is a function of its width; wider teeth can better resist the stresses that are generated when the teeth contact the prey. In this exercise we consider two sharks, the bull shark *Carcharhinus leucas* and the blacktip shark *Carcharhinus limbatus* (Fig. 1). Both species capture small prey whole, but as the size of the prey increases relative to the size of their mouths, they will also employ lateral head shaking to saw out chunks of flesh. The blacktip shark primarily consumes small to medium-sized fish, while the bull shark consumes small to large-sized fish, other sharks, and even sea turtles and mammals.

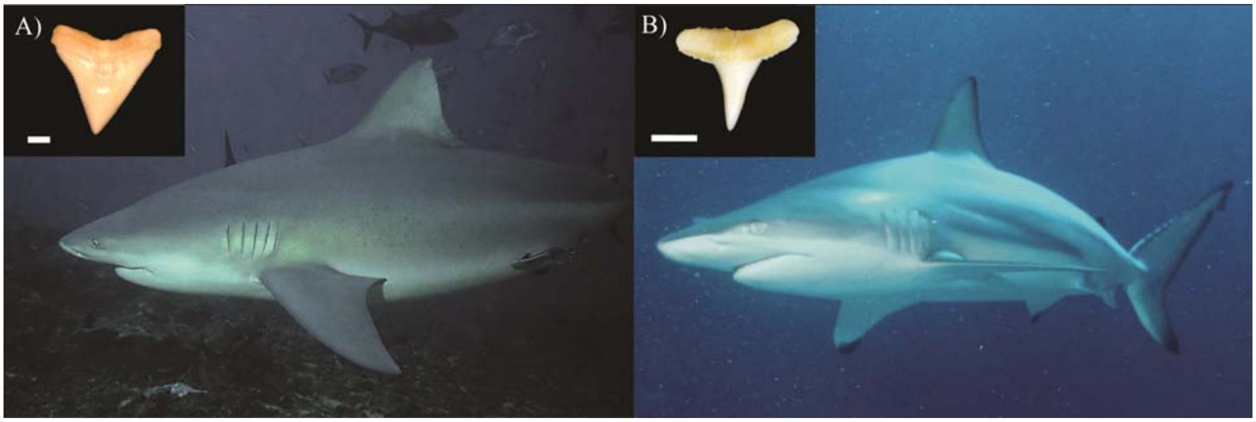

Figure 1 – A) The bull shark *Carcharhinus leucas* with inset of tooth from anterior region of upper jaw. B) The blacktip shark *Carcharhinus limbatus* with inset of tooth from anterior region of upper jaw. Tooth cusps are the more acute triangular areas below the horizontal tooth base. Scale bars indicate 5 mm.

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We begin our analysis with the calculation of tooth pressure. Tooth pressure is equal to bite force divided by the lateral surface area of the tooth in contact with the object being bitten:

$$Tooth\ Pressure = \frac{Bite\ Force}{Tooth\ Lateral\ Surface\ Area}$$

The bull shark tooth below is from a 120 cm shark, while the blacktip shark tooth is from a 152 cm shark (Fig. 2). At these respective lengths, we can assume that the bite force of these two sharks is the same.

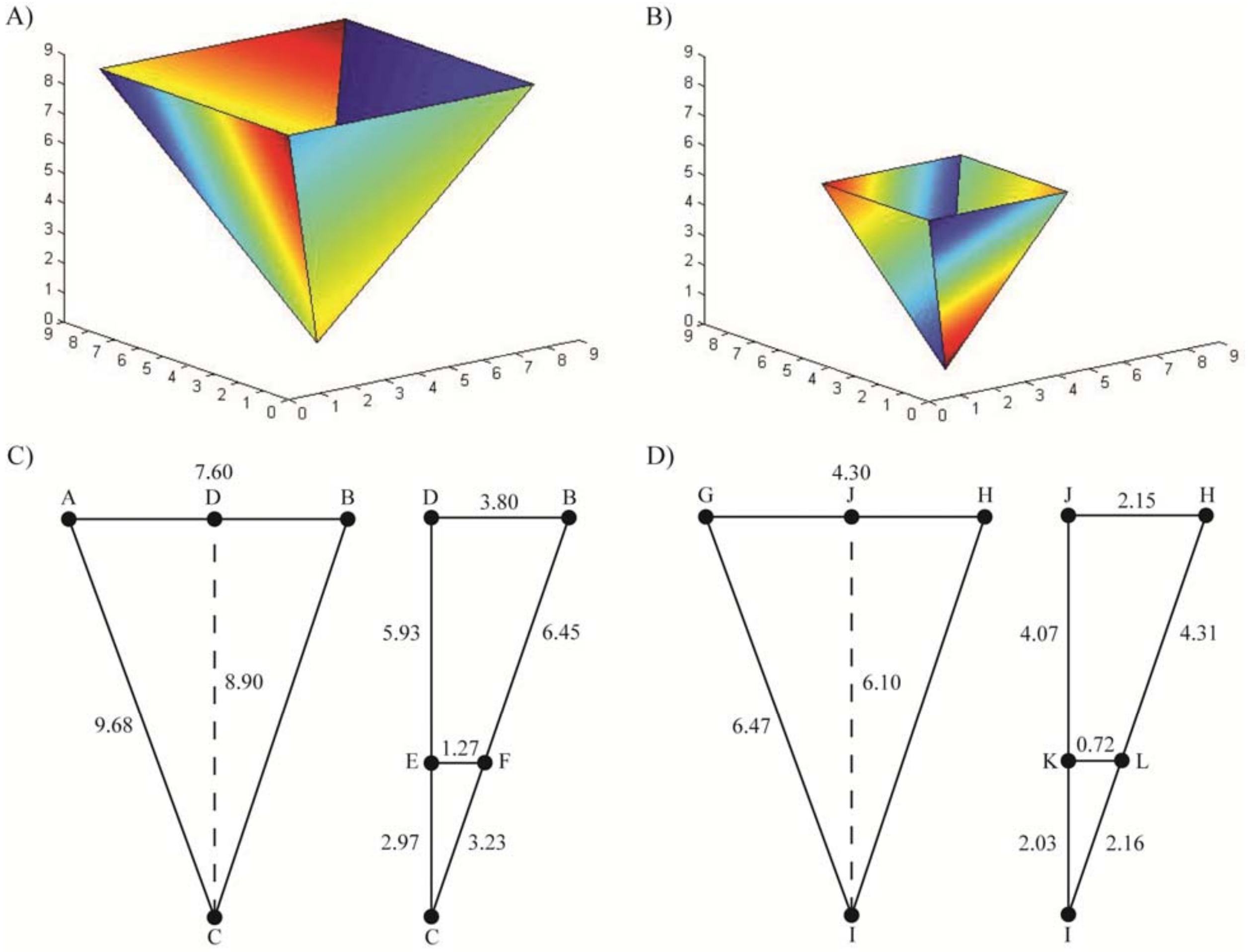

Figure 2 – 3-D representations of bull shark (A) and blacktip shark (B) teeth modeled as right-square pyramids. 2-D cross-sectional representations of bull shark (C) and blacktip shark (D) teeth modeled as right-square pyramids. In C and D the right images are teeth that have been vertically bisected and contain lines parallel to the base at one-third the altitude from the tip of each tooth. Images in A and B were generated with Matlab mathematical software while those in C and D were generated with GeoGebra freeware.

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1. If the bull shark and the blacktip shark have the same bite force, will the tooth with the larger or smaller lateral surface area in contact with the prey item have the greater tooth pressure?

2. Compute the lateral surface area of the bull shark tooth.

3. Compute the lateral surface area of the blacktip shark tooth.

4. In the first model of figure (C), which shows triangle CAB, CD is a perpendicular bisector of AB. In the second model, half of triangle CAB is shown.
   In triangle CDB, EF is parallel to DB. The length of CE is 1/3 the length of CD. Label ED, EF, CF, FB and DB with the correct length for each.

5. Similarly, in figure (D), triangle IJH is half of triangle IGH, with IJ a perpendicular bisector of GH. In triangle IJH, KL is parallel to JH. The length of IK is 1/3 the length of IJ. Label KJ, KL, IL, LH and JH with the correct length for each.

6. Compute the lateral surface area of the bull shark tooth at 1/3 the altitude from the tip.

7. Compute the lateral surface area of the blacktip shark tooth at 1/3 the altitude from the tip.

8. Which tooth has greater lateral surface area, the bull shark tooth or the blacktip shark tooth?

9. Which tooth is better for puncture?

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We now turn our attention to lateral head-shaking. The shark tooth that will be better at sawing through prey is the one better able to resist the forces applied to the side of the tooth. The ability to resist these forces is determined, in part, by the distance from the side of the tooth to the axis running down the middle of the tooth at a particular altitude. This is represented in the above model at 1/3rd the altitude of the tooth by line segment EF for the bull shark tooth and line segment KL for the blacktip shark tooth.

1. What is the measurement of line segment EF?

2. What is the measurement of line segment KL?

3. Which tooth is better for sawing through prey during lateral head-shaking?