Math and Nature

* The dawn of the scientific age coincided with the realization that nature can be described through math. As stated by Galileo in 1623, the entire universe is written in the language of mathematics. Since this time, quantitative analysis of natural phenomena has been at the heart of scientific inquiry. Owing to this relationship between natural phenomena and its description via mathematics, nature provides a tangible context for math that can be used to invite students into math instruction. Nature provides context, which is essential to developing interest in a subject. As educators, it is our responsibility to generate this interest.

The Importance of Context

* In writing, context is the setting in which a story occurs. In many ways, the setting gives meaning to the details of the story and is therefore essential to the development of the plot. As educators we are, in a sense, storytellers that can use context to invite students into the material we are teaching about. Because of this, we believe that many of the components of good story-telling can be used to improve student engagement and knowledge in the classroom.
  + A lack of context is one of the more common complaints about math education because students are always wondering why the mathematical topic is relevant. Context establishes relevance.
* Pedagogical research has shown that creating context for information allows students to relate their own experiences to the material, facilitating experiential and associative learning. This was described by Merrill (2002):
  + First Principles of Instruction:
    - The demonstration principle: Learning is promoted when learners observe a demonstration.
    - The activation principle: Learning is promoted when learners activate prior knowledge or experience.
    - The application principle: Learning is promoted when learners apply the new knowledge.
    - The task-centered principle: Learning is promoted when learners engage in a task-centered instructional strategy.
    - The integration principle: Learning is promoted when learners integrate their new knowledge into their everyday world.
  + The demonstration of new knowledge within a relevant context activates students’ prior knowledge of the subject, which engages them in the material at hand. Once engaged, students are given the opportunity to apply this new knowledge in a task-centered (problem-solving) manner. Association of new knowledge with prior knowledge reinforces this material in the memory, allowing integration of it into new experiences. Integration of knowledge into new experiences is the essence of problem-solving.

Geometry and Biology

* The use of nature as a context for math is extremely evident with regard to biology and geometry. Biological structures vary greatly in their geometry. Given that differences in the geometry of biological structures affect the function of those structures, biology provides a mechanism for geometric problem solving and for illustrating the consequences of geometric variability.
  + The consequences of geometric variability establish the functional aspect of biology, which is what really invites students into the problem. Given the extensive diversity of organisms, their function can provide context for most any math topic.
* A convenient example of the connection between biology and geometry can be found in a bug’s legs, which are almost always cylindrical. Cylindrical structures are good at resisting forces from every direction, which is necessary in the unpredictable natural world. The directions of forces are more controlled in the manmade world, which is why structural elements such as I-beams are designed to resist forces acting in one direction (downward force of gravity).

Shark Teeth

* Shark teeth represent a great case study in which to explore the relationship between biology and geometry. Shark teeth vary greatly in their geometry, and this variability in form determines their cutting performance. Variability in cutting performance affects their feeding ecology, so causality can be established between geometric variability and the ability of sharks to gather resources from their environment.
  + Establishing relevance is essential for engaging your audience, and what kid in the state of FL isn’t familiar with sharks!
  + Slide 5 Figure Key: A) nurse shark, B) tiger shark, C) sand tiger shark, D) mako shark, E) sandbar shark (upper), F) sandbar shark (lower), G) kitefin shark (upper), H) kitefin shark (lower)
  + Slide 9 Figure Key:
    - Top row, left to right: bull shark, blacktip shark, bamboo shark, lantern shark 1, lantern shark 2
    - Bottom row, left to right: sevengill shark, horn shark, lemon shark, great hammerhead shark, spiny dogfish
* One of the things that make shark teeth a great example for illustrating geometry is the relationship between tooth geometry and behavior.
  + Puncturing the teeth into their prey is only one part of the feeding behavior of sharks. Many sharks will follow tooth puncture with a behavior known as “lateral head-shaking” during which the head is rapidly shaken from side to side. This applies lateral forces to the teeth.
    - The embedded video shows a juvenile lemon shark *Negaprion brevirostris* using lateral head-shaking during prey capture.
  + This behavior is an opportunity to further explore the consequences of geometry because what makes one tooth good at puncture might not make it good at tearing laterally through their food, and vice-versa.

Which Tooth is Better for Puncture?

* Determining which tooth is better at puncture is a question of lateral surface area. For a given bite force applied to the tooth, whichever tooth has a smaller lateral surface area will generate greater pressure at the point of application and puncture more easily.
  + Modeling a shark tooth as a right square pyramid provides a convenient way to perform these calculations.
* Our examples for this question are the teeth of a blacktip shark *Carcharhinus limbatus* and a bull shark *Carcharhinus leucas*, both of which are abundant in tropical and subtropical oceans around the world. Both species capture small prey whole, but as the size of the prey increases relative to the size of their mouth they will also employ lateral head-shaking to excise chunks of flesh. The blacktip shark primarily consumes small to medium-sized fish, while the bull shark consumes small to large-sized fish, other sharks, and even sea turtles and mammals.
* Assuming equal bite forces, the series calculations for determining lateral surface area indicates that the blacktip shark tooth is better at puncturing prey items than the bull shark tooth.

Which Tooth is Better for Lateral Head-Shaking?

* Once the teeth puncture into a prey item, many sharks use lateral head-shaking to tear through the food and rip out a chunk. The performance of the teeth during this behavior is again determined by geometry. This time the relevant geometric parameter is one which prevents the teeth from breaking as there are drawn laterally through the prey item.
  + The resistance of any object to bending/breaking is determined by the distribution of material relative to its neutral axis. The neutral axis is an imaginary line that runs through the middle of a structure. When a force is applied to that structure, everything on one side of the neutral axis will be in tension and everything on the other side will be in compression.
  + The distribution of material relative to this neutral axis in the direction of loading is known as the object’s “second moment of area”. Objects have a greater second moment of area, and greater resistance to bending/breaking, if their structural material is distributed at a distance from the neutral axis.
    - Example: meter stick
      * When the meter stick is oriented vertically and force is applied vertically, most of the material is distributed along the axis of loading and the stick does not bend much.
      * When the meter stick is oriented horizontally and force is applied vertically, most of the material is not distributed along the axis of loading and the stick bends considerably.
* The second moment of area of a shark tooth can be approximated by the distance from the lateral margin of the tooth to the neutral axis running down the middle of the tooth. This is a great example because students can actually measure these geometric characteristics if given large enough teeth. Alternatively, these distances can be derived geometrically by determining the length of the base of the right square pyramid at a particular distance along the tooth.
* The series calculations for determining the distance from the lateral margin of the tooth to the neutral axis indicates that the bull shark tooth is better at sawing through prey items during lateral head-shaking than the blacktip shark tooth.