Math and Nature

* The dawn of the scientific age coincided with the realization that nature can be described mathematically. As stated by Galileo in 1623, the entire universe is written in the language of mathematics. Since this time, quantitative analysis of natural phenomena has been at the heart of scientific inquiry. Owing to this relationship between natural phenomena and its description via mathematics, nature provides a tangible context for math that can be used to invite students into math instruction. Establishing context is essential for developing student interest and comprehension of a topic, and for promoting long-term problem solving skills.

The Importance of Context

* In writing, context is the setting in which a story occurs. In many ways the setting gives meaning to the details of the story and is therefore essential to the development of the plot. As educators we are, in a sense, storytellers that can use context to invite students into the material of our classes. Because of this, we believe that many of the components of good story-telling can be used to improve student engagement and knowledge in the classroom.
	+ A lack of context is one of the more common complaints about math education because students are always wondering why a particular mathematical topic is relevant. Context establishes relevance, thereby increasing student interest.
* Pedagogical research has shown that creating context allows students to relate current material to their prior experiences, thereby facilitating experiential and associative learning. This was described by Merrill (2002):
	+ First Principles of Instruction:
		- The demonstration principle: Learning is promoted when learners observe a demonstration.
		- The activation principle: Learning is promoted when learners activate prior knowledge or experience.
		- The application principle: Learning is promoted when learners apply the new knowledge.
		- The task-centered principle: Learning is promoted when learners engage in a task-centered instructional strategy.
		- The integration principle: Learning is promoted when learners integrate their new knowledge into their everyday world.
	+ The demonstration of new knowledge within a relevant context activates students’ prior knowledge of the subject, which engages them in the material at hand. Once engaged, students are given the opportunity to apply this new knowledge in a task-centered (problem-solving) manner. The association of new knowledge with prior knowledge reinforces the material in memory, thereby allowing learned concepts to be applied to new experiences. Integration of knowledge into new experiences is the essence of problem-solving.

Geometry and Biology

* The use of nature as a context for math is extremely evident with regard to biology and geometry. Biological structures vary greatly in their geometry. Given that differences in the geometry of biological structures affect the function of those structures, biology provides a mechanism for geometric problem solving and for illustrating the consequences of geometric variability.
	+ The consequences of geometric variability establish the functional aspect of biology, which is what really invites students into the problem. Given the extensive diversity of organisms, their function can provide context for most any math topic.

Muscle Performance

* Muscle performance is determined by a combination of muscle geometry and physiology. Assuming that the physiological aspect is constant, we can explore the tremendous diversity of muscle geometry as it relates to various tasks and behaviors that animals accomplish. Ultimately, thick muscles generate high forces, long, thin muscles generate fast contractions, and the connections of muscle to skeleton influence the magnitude of both force and power generation.
* Magnitude of force production
	+ The magnitude of force produced by a muscle is proportional to its cross-sectional area. Therefore, those animals that must accomplish tasks requiring great force must have comparably thick muscles. Classic examples of this come from animals that eat things which are difficult to consume. For example, white sharks eat seals which have fairly thick bones. Doing so requires high bite forces, which have been estimated at over 4,000 lbs. In fact, an extinct relative of the white shark (*Megalodon*) is estimated to have had a bite force of nearly 41,000 lbs., which it used to eat whales!
* Frequency of force production
	+ While thick muscles generate a lot of force, they do not contract very quickly. To contract quickly muscles must be long and have a small cross-sectional area. For example, toadfish have a “sonic muscle” which can contract at over 2,000 times per second. This extremely fast contraction is used to generate sounds that are involved in communication and behavioral interactions among toadfish (defending territories, attracting mates).
* Amplification of force production
	+ The force generated by muscles of any size can be amplified when energy is stored in portions of the skeleton. Energy storage in the skeleton is just like stretching a rubber band… when the rubber band is released the energy that was stored during the stretch is released, causing the rubber band to snap back to its original shape. This trick is used by lots of animals to amplify force and save energy. For example, mantis shrimp use their arms like clubs to smash hard prey items like clams. To do so, they first contract a muscle that deforms the shell of their arm, storing energy in the shell. This energy is then released when another muscle contracts and releases the energy stored in the shell. In this manner, mantis shrimp are able to amplify the force generated by their claw muscles to such an extent that they can generate forces 1000X their body weight. Pound for pound, mantis shrimp generate the highest forces in the animal kingdom.
* Power generation
	+ Animals that perform very difficult tasks must generate lots of power to do so. For example, the power generated by bird flight muscles is 10X that of fish swimming muscles, because flying is more difficult than swimming. The most challenging flight behavior of all is hovering, which is the equivalent of taking off and landing over and over again. Therefore, those tiny little hummingbirds are the heavy-weights of power generation. Power is based on both the force produced and speed of movement, so it requires an optimal muscle cross-sectional area and length.

Muscle Modeling

* Muscle force is proportional to cross-sectional area
	+ Muscle force can be estimated based on the cross-sectional area of a muscle, along with its specific tension (Ts) and fiber angle (Θ).
		- Cross-sectional area can be derived geometrically, as will be performed in this exercise.
		- Specific tension (Ts) is a physiological constant that represents the amount of force generated per unit area in a muscle.
		- Muscle fiber angle (Θ) must be accounted for because some of the forces generated by individual muscle fibers cancel each other out. Muscle fibers tend to insert onto a tendon that runs down the middle of the muscle. If the fibers insert at an angle, a portion of the force produced by the muscle will do mechanically useful work (force running along the central tendon of the muscle), while the remainder of the force does not (force vector components from right and left halves of muscle cancel each other out). The mechanically useful portion of the muscle force is accounted for by using the cosine of Θ.
* Muscle force is transmitted to the environment by bones
	+ Muscle force isn’t the only thing that determines how the musculoskeletal system performs. Muscles pull on bones, and it is the leverage of those bones at joints which determines the amount of muscle force that is transmitted to the environment (e.g., biting, running, throwing).
	+ Leverage is measured via “mechanical advantage”, which is the ratio of the in-lever and out-lever in a moving joint.
		- In-lever (Li)
			* Distance from point of rotation in joint to point of muscle insertion
				+ Ex: distance from point of rotation of elbow to tip of elbow
		- Out-lever (Lo)
			* Distance from point of rotation in joint to point at which force is applied into the environment
				+ Ex: distance from point of rotation of elbow to wrist
	+ Multiplying the force produced by the muscle (Fi) and the mechanical advantage of the joint ultimately determines the amount of muscle force transmitted by the skeleton to the environment (Fo). These output forces are what ultimately determine the level of performance when animals do things like bite, dig, run, swim, and fly.
		- $F\_{o}=F\_{i}×\frac{L\_{i}}{L\_{o}}$
* Triceps lever system
	+ The purpose of this exercise is to model the amount of force produced by the triceps lever system in anyone that wants to participate. This system consists of the triceps muscle on the back of the upper arm, the elbow, and the forearm. To do so, each participant must take the following measurements of their arm:
		- Circumference of upper arm (C) at its widest point
		- Length of upper arm (L)
			* Distance from tip of shoulder to tip of elbow
		- Width of tip of elbow (W)
			* Distance across the knob at the tip of the elbow (a.k.a. olecranon process of the ulna)
		- In-lever length (Li)
			* Use the thumb and index finger to feel either side of the elbow. Move the elbow back and forth until you feel the point of rotation on either side of the joint. Measure the distance from the point of rotation to tip of elbow.
		- Out-lever length (Lo)
			* Repeat the process described in the last step and measure the distance from the point of rotation to the wrist.
	+ Procedure
		- Calculate triceps cross-sectional area (CSAt) (Steps 1 – 3)
			* The cross-sectional area of the triceps will be estimated based on the volume of muscle, which will be modeled as a pair of truncated cones that share their bases at the middle of the upper arm. The volume of the lower cone will be estimated and then doubled to account for the upper cone.
			* The circumference of the upper arm (C) is used to determine the diameter (Darm) and radius (Rarm) of the arm, from which the radius of triceps (Rt) is determined. This value is the radius of the bottom of the cone.
			* The volume of the truncated cone is estimated by subtracting the volume of the truncated portion from the volume of the full cone, which requires a value for the height (H) of the full cone. This value is determined using principles of similar triangles where the radius of the triceps (Rt), length of the upper arm (L), and width of the tip of the elbow (W) have already been determined.
				+ $\frac{H}{R\_{t}}=\frac{H-\frac{L}{2}}{\frac{W}{2}}$
				+ The volume of the truncated cone (V) is then determined by subtracting the volume of the truncated portion from the volume of the entire cone. This volume is then multiplied by 2 to determine the volume of both cones included in the triceps model (Vt).

$V=\frac{1}{3}π\left[\left(R\_{t}^{2}H\right)-\left(\left(\frac{W}{2}\right)^{2}\left(H-\frac{L}{2}\right)\right)\right]$

$V\_{t}=2×V$

* + - * Muscle fiber length (FLt) must then be estimated by multiplying the radius of the triceps (Rt) by $\sqrt{2}$. $ \sqrt{2} $can be used because it was assumed that the muscle fiber angle of the triceps is 45°. Therefore, the muscle can be modeled as a 45-45-90 right triangle. Alternatively, if the fiber angle were not 45°, fiber length could be solved using trigonometry by dividing the radius of the triceps (Rt) by the sine of the muscle fiber angle (Θ).
				+ $FL\_{t}=\frac{R\_{t}}{\sin(θ)}=R\_{t}\sqrt{2}$
			* The cross-sectional area of the triceps is then determined by dividing the volume of the triceps (Vt) by its fiber length (FLt). This is done because a cubic function (muscle volume) divided by a linear function (fiber length) results in a square function (muscle area).
				+ $CSA\_{t}=\frac{V\_{t}}{FL\_{t}}$
		- Calculate triceps muscle force (Ft) (Step 4)
			* Multiply the triceps cross-sectional area (CSAt) by the specific tension of mammalian muscle (Ts = 51 lb/in2) and the cosine of the fiber angle of the triceps muscle (Θ = 45°). These are average values based on human muscle physiology and geometry.
			* $F\_{t} =(CSA\_{t}×T\_{s})×\cos(θ)$
		- Calculate triceps lever system force (Ftls) (Step 5)
			* Multiply the force produced by the triceps muscle (Ft) by the leverage of the elbow (Li/Lo).
			* $F\_{tls}=F\_{t}×\frac{L\_{i}}{L\_{o}}$
		- This final value (Ftls) should be approximately the amount of weight that each participant can generate doing a reverse curl (elbow flexion) with one arm.