**Cone Volume Activity Notes**

**Calculus Connection: Cone Formula**

1. This note explains the precise derivation of the formula for a cone by rotating a line segment about the x-axis.

<http://mathforum.org/library/drmath/view/51718.html>

**Details:** Create a cone by rotating a line about the x-axis. Let the radius of the base of the cone be **a**, and the height of the cone be **h**. Then the line which will be rotated is . (Drawn with GeoGebra)

 

To find the exact volume of the cone, start with the volume of one slice of the cone perpendicular to the x-axis of width. For small this volume will be approximately, since the radius of an arbitrary slice is . Next, integrate this formula from 0 to h (using the Fundamental Theorem of Calculus to move from a sum of slices to the exact volume), to capture the volume of the entire cone.

**Calculus Formula: Pyramid Formula**

1. This note explains the precise derivation of the formula for a pyramid using the three-dimensional expansion of the technique of estimating the area under a curve in two-dimensional space with rectangles and taking the limit as the number of rectangles of uniform width goes to infinity.

<http://mathforum.org/library/drmath/view/53646.html>

 

Consider this structure, which is a pyramid made up of three steps. The top step has side length and the height is . The volume of this step is .

The next step has four blocks the same size as the top step and the bottom step has nine.

The formula for the volume of the entire shape is .

 

Now consider a refinement of the previous pyramid. This step-pyramid has six steps, with the same base area as the previous pyramid, and the same height.

The volume of the top step is .

The total volume of the figure is .

If we continue this refinement, a pyramid with steps and a top block (the building block) with volume , will have a total volume of

.

Important Note: As increases, both the length of the standard building block, and its height, decrease. However, the base area and the height are fixed throughout the refinement process.

The area of the base of the pyramid is regardless of the number of steps .

(i.e. The building blocks have a base area of and there are building blocks on the bottom level.)

The height of the pyramid is regardless of the number of steps .

It is known that . (Recall the story of Gauss adding the numbers from 1 to 100 when he was in elementary school by developing a formula.)

Our total volume formula may be transformed as follows:

Now we take the limit as goes to infinity, which mathematically represents the process of continuing to refine our step pyramid until it morphs into a smooth pyramid with a square base and triangular sides.