Math and Nature

* The dawn of the scientific age coincided with the realization that nature can be described through math. As stated by Galileo in 1623, the entire universe is written in the language of mathematics. Since this time, quantitative analysis of natural phenomena has been at the heart of scientific inquiry. Owing to this relationship between natural phenomena and its description via mathematics, nature provides a tangible context for math that can be used to invite students into math instruction. Nature provides context, which is essential to developing interest in a subject. As educators, it is our responsibility to generate this interest.

The Importance of Context

* In writing, context is the setting in which a story occurs. In many ways, the setting gives meaning to the details of the story and is therefore essential to the development of the plot. As educators we are, in a sense, storytellers that can use context to invite students into the material we are teaching about. Because of this, we believe that many of the components of good story-telling can be used to improve student engagement and knowledge in the classroom.
  + A lack of context is one of the more common complaints about math education because students are always wondering why the mathematical topic is relevant. Context establishes relevance.
* Pedagogical research has shown that creating context for information allows students to relate their own experiences to the material, facilitating experiential and associative learning. This was described by Merrill (2002):
  + First Principles of Instruction:
    1. The demonstration principle: Learning is promoted when learners observe a demonstration.
    2. The activation principle: Learning is promoted when learners activate prior knowledge or experience.
    3. The application principle: Learning is promoted when learners apply the new knowledge.
    4. The task-centered principle: Learning is promoted when learners engage in a task-centered instructional strategy.
    5. The integration principle: Learning is promoted when learners integrate their new knowledge into their everyday world.
  + The demonstration of new knowledge within a relevant context activates students’ prior knowledge of the subject, which engages them in the material at hand. Once engaged, students are given the opportunity to apply this new knowledge in a task-centered (problem-solving) manner. Association of new knowledge with prior knowledge reinforces this material in the memory, allowing integration of it into new experiences. Integration of knowledge into new experiences is the essence of problem-solving.

Geometry and Biology

* The use of nature as a context for math is extremely evident with regard to biology and geometry. Biological structures vary greatly in their geometry. Given that differences in the geometry of biological structures affect the function of those structures, biology provides a mechanism for geometric problem solving and for illustrating the consequences of geometric variability.
  + The consequences of geometric variability establish the functional aspect of biology, which is what really invites students into the problem. Given the extensive diversity of organisms, their function can provide context for most any math topic.
* Ecological interactions can be affected by the geometry of anatomical structures. This idea is inherent to biology because, for example, animals with large mouths can eat animals with small bodies and animals with long limbs can outrun those with short limbs. Geometric variability among closely related species is often thought to be evidence of past competition in which two organisms (or species) were competing for the same resource. Based upon the Principle of Competitive Exclusion, no two organisms can occupy the same ecological niche (role in the environment encompassing resources utilized and interactions with the environment). One of these organisms (or species) is bound to have some slight advantage that results in it out-competing the other. Having been out-competed, the other organism must switch to another set of resources which, over time, may result in the evolution of some alternate geometry better suited for these other resources.
  + Mouth size is an extremely important geometric parameter in this regard. Predators with large mouths have a competitive advantage because they are able to consume larger prey, which provides a greater energetic return per unit effort than small prey. Therefore, large mouthed predators can displace small mouthed predators from a particular ecological niche.

Otter vs. Pufferfish

* The relationship between mouth size and predatory ability is generally clear cut. The larger the predator’s mouth, the larger the prey it can eat. However, an interesting case arises when we examine prey items that are able to change size when approached by a predator, as is the case with pufferfish.
  + Embedded video: River otter attempting to capture an inflated pufferfish. The video can be viewed at <http://www.juzztv.com/watch_video.php?v=a492c027e4eb8c2>.
* Pufferfish have evolved the ability to increase in size when threatened by a predator by pumping water into an expandable stomach. This process is aided by extensible skin and the absence of ribs.
* The objective of this exercise is to determine the largest pufferfish that an otter with jaws of a certain size can eat. The otter’s jaws are modeled as tangents to the circle representing the pufferfish, and it is assumed that the jaws must be tangent in order for the otter’s teeth to puncture into the pufferfish. The “gape angle” represents the largest angle to which the otter can open its jaws.