Math and Nature

* The dawn of the scientific age coincided with the realization that nature can be described mathematically. As stated by Galileo in 1623, the entire universe is written in the language of mathematics. Since this time, quantitative analysis of natural phenomena has been at the heart of scientific inquiry. Owing to this relationship between natural phenomena and its description via mathematics, nature provides a tangible context for math that can be used to invite students into math instruction. Establishing context is essential for developing student interest and comprehension of a topic, and for promoting long-term problem solving skills.

The Importance of Context

* In writing, context is the setting in which a story occurs. In many ways the setting gives meaning to the details of the story and is therefore essential to the development of the plot. As educators we are, in a sense, storytellers that can use context to invite students into the material of our classes. Because of this, we believe that many of the components of good story-telling can be used to improve student engagement and knowledge in the classroom.
  + A lack of context is one of the more common complaints about math education because students are always wondering why a particular mathematical topic is relevant. Context establishes relevance, thereby increasing student interest.
* Pedagogical research has shown that creating context allows students to relate current material to their prior experiences, thereby facilitating experiential and associative learning. This was described by Merrill (2002):
  + First Principles of Instruction:
    - The demonstration principle: Learning is promoted when learners observe a demonstration.
    - The activation principle: Learning is promoted when learners activate prior knowledge or experience.
    - The application principle: Learning is promoted when learners apply the new knowledge.
    - The task-centered principle: Learning is promoted when learners engage in a task-centered instructional strategy.
    - The integration principle: Learning is promoted when learners integrate their new knowledge into their everyday world.
  + The demonstration of new knowledge within a relevant context activates students’ prior knowledge of the subject, which engages them in the material at hand. Once engaged, students are given the opportunity to apply this new knowledge in a task-centered (problem-solving) manner. The association of new knowledge with prior knowledge reinforces the material in memory, thereby allowing learned concepts to be applied to new experiences. Integration of knowledge into new experiences is the essence of problem-solving.

Geometry and Biology

* The use of nature as a context for math is extremely evident with regard to biology and geometry. Biological structures vary greatly in their geometry. Given that differences in the geometry of biological structures affect the function of those structures, biology provides a mechanism for geometric problem solving and for illustrating the consequences of geometric variability.
  + The consequences of geometric variability establish the functional aspect of biology, which is what really invites students into the problem. Given the extensive diversity of organisms, their function can provide context for most any math topic.

Suction Feeding

* Vertebrate Skulls
  + The great diversity of skull morphology amongst vertebrates is associated with the varied food resources that they consume in aquatic and terrestrial habitats. During the course of evolutionary history there has been a trend towards the reduction of bones and mobility in the vertebrate skull, resulting in more stable structures capable of generating greater force during feeding. This has particularly been the case among the terrestrial vertebrates. Conversely, fish skulls are highly kinetic and composed of numerous moving parts.
    - The average fish skull has about 50 bones with at least 7 moving parts.
    - The human skull has only 22 bones and a single moving part (i.e., lower jaw).
  + The anatomy and mobility of the fish skull enables a feeding behavior known as “jaw protrusion” (see videos of sand tiger shark and sling-jaw wrasse in presentation), during which the upper jaw slides forward while the lower jaw is depressed, allowing the predator to:
    - Generate suction pressure to draw food into the mouth
    - Increase the speed of prey capture
    - Capture prey items from a distance
* Fish Feeding Mechanisms
  + All fish use one of the following feeding mechanisms to capture prey.
    - Filter feeding
      * Filter feeding fish, such as the whale shark, have highly modified gill structures that help capture planktonic prey. The gills have pads like sieves which trap prey while the fish swim forward with their mouths open.
    - Biting
      * Biting involves the use of the jaws to capture and process prey, and is usually associated with the consumption of hard, benthic prey that has limited escape ability. Accordingly, fish that use biting usually have forceful, high leverage jaws with pavement like teeth for crushing and grinding food.
        + Examples:

Horn sharks *Heterodontus francisci* eats sea urchins from the sea floor.

Parrotfish *Sparidae spp.* crush chunks of coral and digest the microorganisms living on the coral.

* + - Ram feeding
      * Ram feeding involves the predator overtaking the prey by swimming faster than it can escape. It is usually associated with fish that have fast jaws and sharp teeth.
        + Examples:

White shark *Carcharodon carcharias*

Great barracuda *Sphyraena barracuda*

* + - Suction feeding
      * Suction feeding is the most common feeding mechanism and is believed to have been the original fish feeding mechanism that originated in evolutionary history.
      * Suction feeding occurs when a fish rapidly expands its mouth and gill region. The increase in volume due to expansion creates a negative pressure that causes water to flow into the mouth, along with any prey items that are located within the moving water. This process is aided by jaw protrusion, which was described earlier.
        + The amount of suction pressure that is generated is based on both the magnitude and rate of volume change. Fish that create a lot of expansion in a short period of time, such as the largemouth bass *Micropterus salmoides*, can capture their prey in less than a tenth of a second.
* Geometric Modeling of Suction Feeding
  + Sometimes the best science happens when researchers find very simple models to explain very complicated events. One such example is suction feeding, in which the head can be modeled as a series of expanding cones. The purpose of this exercise is to use a simple model (1 full cone + 1 truncated cone) of the feeding mechanism of the goliath grouper *Epinephelus itajara* to answer the following questions:
    - What fluid velocity can the goliath grouper generate during suction feeding?
    - How does suction feeding by the goliath grouper compare to other fish?
  + Determine the velocity of water flow into the mouth
    - First the volume of the full and truncated cones must be determined at rest (t0 = time at rest) and at maximum expansion (t1 = time at maximum expansion) based on dimensions provided in the presentation.
    - Then, the cross-sectional area of the mouth must be determined based on the dimensions of the expanded cones.
    - Then, the duration of mouth opening must be determined from the values of t0 and t1.
    - Water flow velocity will then be determined using the equations:
  + Comparison to other fish
    - Perform the same series of calculations on the snook *Centropomus undecimalis* and longjaw butterfly fish *Forcipiger longirostris* to determine which of the thre fish generates the greatest water flow velocity into the mouth.
    - While the snook generates the greatest velocity, note that there are very different strategies all of which can result in high performance suction feeding. The goliath grouper has a tremendous increase in volume, but does so relatively slowly (0.132 s), whereas the longjaw butterfly fish has a tiny volume change, but does so very rapidly (0.022 s).